Cognitive abilities as precursors of the early acquisition of mathematical skills

Habilidades cognitivas como precursoras de la adquisición de las habilidades matemáticas tempranas

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Abstract

Specific role of the domain-general and domain-specific precursors in the development of early mathematics is still under debate. In addition, there is no general agreement about the detection of the main components of domain-specific precursors. The aim of the present work is to get more insight in the role played by a fundamental domain-general precursor, the working memory, in the development of mathematical skills. Other goal for this paper is to propose a general framework for the classification of domain-specific precursors of mathematical learning.

Palabras clave: Approximate number system, early mathematics, mathematical precursors, Object tracking system, Working Memory.

Resumen

La función específica de los precursores de dominio general y específicos en el desarrollo de principios de matemáticas sigue siendo objeto de debate. Además, no existe un acuerdo general acerca de la cuáles son los principales componentes de los precursores de dominio específico. El objetivo del presente trabajo es obtener una visión más clara del papel desempeñado por un precursor de dominio general fundamental, la memoria de trabajo, en el desarrollo de habilidades matemáticas. Otro objetivo es proponer un marco general para la clasificación de los precursores de dominio específico del aprendizaje matemático.

Keywords: sistema numérico aproximado, matemática temprana, precursores matemáticos, sistema de seguimiento de objetos, memoria de trabajo.

The number of students with mathematical difficulties has greatly increased over the last 20 years (Swanson, 2000). Several studies found that between 5% and 10% of children and adolescents experience a substantive learning deficit in at least one area of mathematics (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen; Shalev, Manor, & Gross-Tsur, 2005; Shalev, 2007). These students that find mathematics difficult choose not study math in secondary or further education (Brown, Askew, Millett, & Rhodes, 2003). This choice must be considered a risk factor as several studies found that mathematical abilities predict financial and educational success, particularly for women (Bynner & Parsons, 2006; Geary, Hoard, Nugent, & Bailey, 2013).

An increasing number of studies have investigated the cognitive components that mainly contribute to the development of mathematical skills confirming that some abilities in kindergartners can predict later mathematics achievement outcome (De Smedt et al., 2009; Krajewski & Schneider, 2009b; Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009; Mazzocco & Thompson, 2005; Passolunghi, Vercelloni, & Schadee, 2007; Passolunghi & Lanfranchi, 2012). Competencies that specifically predict mathematical abilities may be considered specific precursors, whereas general cognitive abilities (i.e., domain-general precursors) may predict performance not only in mathematics but also in other school subjects.

However, the specific role of the domain-general and domain-specific precursors in the development of early mathematics is still under debate. In addition, there is no general agreement about the detection of the main components of domain-specific precursors (Berch, 2005; Gersten et al., 2005). The aim of the present work is to get more insight in the role played by a fundamental domain-general precursor, the working memory, in the development of mathematical skills. Moreover, we attempted to propose a general framework for the classification of domain-specific precursors of mathematical learning.

Working Memory Abilities and Mathematics

With regard to domain-general precursors, some general cognitive abilities, such as working memory, processing speed, and intelligence level, predict performance in mathematics (De Smedt et al., 2009; Espy et al., 2004; Passolunghi & Lanfranchi, 2012; Passolunghi, Mammarella, & Altoè, 2008; Passolunghi et al., 2007). The role of the domain-general precursors of learning is particularly important during preschool years, but their involvement seems to decrease in the following years as a consequence of a greater influence of domain-specific abilities (Passolunghi & Lanfranchi, 2012). Among these general cognitive abilities, several studies demonstrated that working memory is a main predictor of mathematical competence (De Smedt et al., 2009; Gathercole & Pickering, 2000; Gersten, Jordan, & Flojo, 2005; Jordan, Kaplan, Nabors Oláh, & Locuniak, 2006; Krajewski & Schneider, 2009b; Passolunghi & Lanfranchi, 2012). The term "working memory"(WM) refers to a temporary memory system that plays an important role in supporting learning during the childhood years because its key feature is the capacity to both store and manipulate information (Bull & Scerif, 2001; Gathercole & Alloway, 2006; Miyake & Shah, 1999). Indeed, different mathematical tasks, such as performing mental arithmetic and understanding mathematical word problems, require the storage of information while it is processed or integrated with information retrieved from long-term memory (Swanson, 2004; Tronsky, 2005). Furthermore, WM skills are necessary even when very young children need to mentally represent and manipulate quantitative information (Alibali & DiRusso, 1999).

Further evidence of the importance of working memory in children's mathematical skills has been provided by longitudinal studies that demonstrated that working memory performance in preschoolers predicts mathematical achievement several years after kindergarten (Gathercole, Brown, & Pickering, 2003; Mazzocco & Thompson, 2005; Passolunghi & Lanfranchi, 2012). Specifically, several studies showed a direct influence of working memory on mathematical achievement in first and second graders (De Smedt et al., 2009; Passolunghi et al., 2008, 2007).

Various models of the structure and function of working memory exist, but the present study considered the multicomponent model of working memory proposed by Baddeley and Hitch in 1974 and revised in succeeding years (Baddeley, 1986, 2000). This model consists of three main parts. The two "slave" systems of working memory (i.e., the phonological loop and visual-spatial sketchpad) are specialized for processing language-based and visuo-spatial information, respectively. The central executive, which is not modality-specific, coordinates the two slave systems and is responsible for a range of functions, such as the attentional control of actions. The distinctions between the central executive system and specific memory storage systems (i.e., the phonological loop and visuo-spatial sketchpad) in some ways parallel the distinction between working memory, involving storage, processing, and effortful mental activity, and short-term memory, typically involving situations in which the individual passively holds small amounts of information (Swanson & Beebe-Frankenberger, 2004).

With regard to the contribution of the three core components of working memory to the development of mathematical skills, many studies showed a direct association between executive function and children's early emergence and development of mathematical abilities across a wide age range (Bull, Espy, & Wibe, 2008; Bull & Scerif, 2001; Gathercole, Pickering, Knight, & Stegmann, 2004; Gathercole & Pickering, 2000; McLean & Hitch, 1999). For example, dual task studies suggest that central executive resources are implicated in children's arithmetic performance (e.g. Imbo & Vandierendonck, 2007) and longitudinal data found that inhibitory control predict later math outcomes (Blair & Razza, 2007; Mazzocco & Kover, 2007). On the other and, children who are poor in mathematics have poor performance in central executive tasks, especially in tasks that require the inhibition of irrelevant information and updating (Espy et al., 2004; Passolunghi, Cornoldi, & De Liberto, 1999; Passolunghi & Siegel, 2001; St Clair-Thompson & Gathercole, 2006).

Spatial skills and visuo-spatial working memory were also found to be related to children's early counting ability (Kyttälä, Aunio, Lehto, Van Luit, & Hautamäki, 2003) and general mathematical competence (Jarvis & Gathercole, 2003; Passolunghi & Mammarella, 2011). Indeed, the visuo-spatial sketchpad appears to support the representation of numbers in counting, arithmetic calculations, and especially mental calculation (D'Amico & Guarnera, 2005; Heathcote, 1994; McKenzie, Bull, & Gray, 2003; McLean & Hitch, 1999). This component is also fundamental in the process of problem solving because it allows the individual to build a visual mental representation of the problem (Holmes & Adams, 2006). Moreover, visuo-spatial WM abilities assessed in preschool years predict complex arithmetic, number sequencing and graphical representation of data in primary school (Bull et al., 2008).

The results of studies that considered the role of the phonological loop in children's mathematical processing have been unclear. Dual task studies showed that 8-9 year old children (but not younger children) use a verbal approach supplemented by visual-spatial resources during on-line arithmetic performance (McKenzie et al., 2003). In the field of learning disabilities, some studies have found no differences in phonological loop abilities between children with and without mathematical difficulties, especially when differences in reading ability were controlled (McLean & Hitch, 1999; Passolunghi & Siegel, 2001, 2004). Other authors suggested that the phonological loop is involved in basic fact retrieval (Holmes & Adams, 2006). The role of each working memory component in mathematical cognition must be considered to vary with expertise and development (Meyer, Salimpoor, Wu, Geary, & Menon, 2010), with an increasing involvement of the phonological loop in mathematical cognition from the age of seven onward (Hitch, Halliday, Schaafstal, & Schraagen, 1988; Raghubar, Barnes, & Hecht, 2010; Rasmussen & Bisanz, 2005).

Domain-specific Precursors of Mathematics

Another important aspect of the acquisition of mathematical skills is represented by domain-specific components. Foundational-specific skills necessarily underlie the development of arithmetic skills. The literature identified two abilities that could be considered fundamental domain-specific precursors of mathematical learning: the "number sense" or the ability to represent and manipulate numbers nonverbally (Dehaene, Molko, Cohen, & Wilson, 2004; Dehaene, 1997; Gilmore, McCarthy, & Spelke, 2007), and the early symbolic numeracy (Geary, Hoard, & Hamson, 1999; Passolunghi & Lanfranchi, 2012). Although researchers agree that children develop number sense and early symbolic numeracy abilities prior to the development of formal mathematical skills, they disagree about the precise definition of early numeracy and number sense (Berch, 2005; Gersten et al., 2005). With the presented work, based on previous findings, we tried to propose a general framework for the classification of domain-specific precursors of mathematical learning.

Number sense

No unique definition of number sense exists (Gersten et al., 2005), but it generally includes subitizing (i.e., the rapid and accurate enumeration of small sets of objects), making quantity comparisons, estimating, and forming representations of numerical magnitudes in the form of a mental number line (Berch, 2005; Dehaene, 1997; Jordan et al., 2006). Such skills develop before formal instruction and have been suggested to be innate (Butterworth, 2005; Dehaene, 1997; Jordan et al., 2006), but refinements are also observed as a consequence of teaching and training procedures (Ramani & Siegler, 2008; Van Herwegen, Costa, & Passolunghi, under review; Whyte & Bull, 2008).

In particular, there are two core cognitive systems responsible for non-verbally representing numbers. Indeed, large and small numerosities seems to activate different systems (Feigenson, Dehaene, & Spelke, 2004). The Approximate Number System (ANS) is used for representing large, approximate numerical magnitudes, and the second system (Object Tracking System) is used for the precise representation of small numbers. Both can be observed in adults, infants and other animal species (Feigenson et al., 2004).

The approximate number system

The ANS, is a cognitive system that underlies the preverbal ability to perceive and discriminate approximate large numerosities. This ability is robust across multiple modalities of input (e.g. visual stimuli or sounds stimuli), increases in precision over the development and is ratio dependent according to Weber's law (Barth, Kanwisher, & Spelke, 2003; Halberda & Feigenson, 2008a; Lipton & Spelke, 2003). Within the ANS, numerosities seem to be ordered spatially in a sort of mental number-line with increasing acuity throughout development (Feigenson et al., 2004; C. Gallistel & Gelman, 2000; Halberda & Feigenson, 2008b). On this mental number-line, each numerosity has a specific position with smaller numerosities placed at the left and bigger numerosities placed on the right (Dehaene, Bossini, & Giraux, 1993). Number-line representations of numerical magnitude are logarithmic compressed (Dehaene, 2007; Siegler & Opfer, 2003) such that the perceived distance between small quantities is larger than the perceived distance between big quantities (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007). As a consequence small quantities are overrepresented on the mental number-line.

The approximate number representations can be mentally combined to perform comparison, addition and substraction across sets. ANS can be asessed using tasks which involve viewing, comparing, adding, or subtracting non-symbolic quantities, such as arrays of dots (C. Gilmore, Attridge, & Inglis, 2011; Halberda & Feigenson, 2008a; Piazza, Pinel, Le Bihan, & Dehaene, 2007).

The ANS seems to be active since the first few months of life. Indeed, it has been showed how 6-9 month old infants exhibit the capacity to create abstract representation of numerosity that support the ability to discriminate numerosities, recognize the ordinal relationship between numerosities and form expectation of about the outcomes of simple arithmetic problems (Lipton & Spelke, 2003; McCrink & Wynn, 2004; Xu & Spelke, 2000). The ANS systems remains active also in older children and adults (Barth et al., 2003; Cordes, Gelman, Gallistel, & Whalen, 2001).

ANS has been found to correlate with mathematics achievement (Bonny & Lourenco, 2013; Halberda, Mazzocco, Feigenson, & Halberda, 2008; Libertus, Feigenson, & Halberda, 2011) and it is severely impaired in children with developmental dyscalculia e.g. (Mazzocco, Feigenson, & Halberda, 2011).

It remains unclear when the noisy ANS representation integrate with more formal math abilities. It has been suggest that the acquisition of the meaning of symbolic numerals is done by mapping symbolic numerals (number words or Arabic digits) onto the pre-existing approximate number representation. As a consequence, ANS provides semantic representations of numbers and the precision of the ANS would play a crucial role in the early foundation of symbolic number knowledge (Dehaene, 1997; Dehaene et al. 2013; Holloway and Ansari, 2009; Wynn,1992).

The object tracking system

The Object Tracking System (OTS) is a cognitive system that allow precise representation of distinct individuals. This system involved in keep tracking of small number of objects (up to three-four items) and for representing information about continuous quantitative properties of objects. Similarly to the ANS, the OTS varies across individuals (Halberda et al., 2008; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008) and is subject to maturation (Oakes, Ross-Sheehy, & Luck, 2006; Rose, Feldman, & Jankowski, 2001). It is a non-numerical mechanism that support the quickly, accurately and effortlessly perception of the numerosity of small sets of objects, a phenomenon known as subitizing. This ability to discriminate between small quantities do not depend on numerical ratio but on the absolute number of items presented (Feigenson, Carey, & Hauser, 2002; Feigenson & Carey, 2003), and it is robust across modalities (Wynn, 1996). Moreover, The OTS seems to be fundamental also to compute information about the continuous quantitative properties of stimuli (Clearfield & Mix, 1999; Feigenson et al., 2002; Xu & Spelke, 2000). For example, when 10-12 months old infants are asked to choose between to quantities of crackers they choose the larger quantity with comparisons of 1 versus 2 and 2 versus 3 (but failed with comparisons of 3 versus 4, 2 versus 4, and 3 versus 6). Differently, when crackers were different sizes, total surface area or volume determined the choice. Infants success in this tasks only when 3 or fewer objects were hidden in either location (Feigenson et al., 2002).

The OTS supports visual enumeration of small sets of objects but its role in performing symbolic number tasks is not yet clear (Piazza, 2010). Information about the importance the OTS in numerical development comes from studies in the field of learning disabilities. Indeed, children with developmental dyscalculia have a deficit in subitizing and tend to use serial counting to determine the numerosity of small sets of objects (Landerl, Bevan, & Butterworth, 2004; Moeller, Neuburger, Kaufmann, Landerl, & Nuerk, 2009; Schleifer & Landerl, 2011), a finding that suggests a importance of the OTS for the development of numerical abilities (Carey, 2001; Le Corre & Carey, 2007). Moreover, the exact small number representation seems to be involved even in adults 'symbolic number processing. Adults show an immediate and accurate recognition of numerosities 1-4, after which error rate and response time increase significantly. These results show that small numbers are processed via subitizing , differently from large numbers (Mandler & Shebo, 1982; Pylyshyn, 2001; Trick & Pylyshyn, 1994). In sum, small arrays of objects activate the OTS responsible for representing and tracking numerically distinct individuals. The activation of this system allows to process either continuous quantitative properties of objects, or the number of individuals in the array (Feigenson et al., 2004).

From Number Sense to Early Symbolic Numeracy

The acquisition of early symbolic numeracy, that includes the ability to identify number symbols as well as counting skills, allow children to go beyond the pre-verbal number processing systems. The development of early symbolic numeracy is grounded on the number sense skills which provide semantic representations of numbers (Piazza, 2010). Therefore, during development we can observe the creation of a connection between quantities and symbolic numbers, providing the number symbols with a non-symbolic magnitude meaning (Dehaene, 2001; Hannula & Lehtinen, 2005). Some studies found that in this process the ANS plays a fundamental role (Dehaene & Changeux, 1993; Gallistel & Gelman, 1992; Lipton & Spelke, 2003) others stressed the importance of OTS (Carey, 2001, 2009; Le Corre & Carey, 2007) whereas other studies considered the combination of the two systems as crucial (Feigenson et al., 2004; Spelke & Kinzler, 2007). The link between symbolic and non-symbolic representations of number has been called number sense access (Rousselle & Noël, 2007; Wilson, Dehaene, Dubois, & Fayol, 2009) and is an important skill for math learning

(Kolkman, Kroesbergen, & Leseman, 2013). Indeed, the integration of the ANS and the OTS through verbal counting, seems to pave the way to the understanding of exact number (Carey, 2001; Le Corre & Carey, 2007; Lipton & Spelke, 2003).

At 2-3 years old children learn to count and thereby already acquire precise number words. However, this early counting list is numerically meaningless and they do not yet use number words to describe quantities (Fuson, 1988; Krajewski & Schneider, 2009a; Le Corre, Van de Walle, Brannon, & Carey, 2006; Wynn, 1990, 1992). The numerals in the list function as placeholders that can be mapped onto core representations of numbers to support the acquisition of the counting principles (Le Corre & Carey, 2007). Then, children gradually learn that "4" matches an array of four objects and that the number "20" is bigger than number "7".

Adults and young children access non-symbolic representations of numbers when solving problems presented in Arabic or verbal form (Gilmore, McCarthy, & Spelke, 2007). However, the automaticity of the connection from symbols to quantities is not yet established in early childhood (Girelli, Lucangeli, & Butterworth, 2000; Rousselle & Noël, 2007) and became gradually automatic with development (Naccache & Dehaene, 2001; Rusconi, Priftis, Rusconi, & Umilta, 2006). Thus, children quickly learn to map symbolic numbers onto their preexisting number-line representation of numerical magnitude. This mapping that is initially logarithmic becomes linear during development as people learn to compensate for the logarithmic compression of the mental number line .

In the development of mathematical abilities, non-symbolic quantity skills, symbolic skills as well as development of accurate number sense access are found to be important for learning more advanced math operation such as addition or subtraction (Booth & Siegler, 2006; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Kolkman et al., 2013). However recent studies highlighted the fundamental role played by the number sense access for mathematical achievement and arithmetic strategy use during development (De Smedt & Gilmore, 2011; Rousselle & Noël, 2007; Vanbinst, Ghesquière, & De Smedt, 2012) suggesting that number sense access could be an alternative core deficit in dyscalculia (Rousselle & Noël, 2007; Rubinsten & Henik, 2005).

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